

**Attendance Questions: Find the area of each shape. Round your answer to the nearest tenth.**

1. An equilateral triangle with edge length 20 cm.

2. A regular hexagon with edge length 14 m

3. A circle with radius 6.8 in

4. A circle with diameter 14 ft

- I can learn and apply the formula for the volume of a prism.
- I can learn and apply the formula for the volume of a cylinder.

### Common Core

**CC.9-12.G.GMD.3** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

**CC.9-12.G.GMD.1** Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.

**CC.9-12.G.MG.1** Use geometric shapes, their measures, and their properties to describe objects.

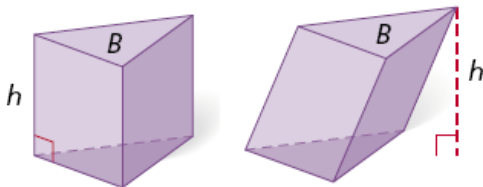
**CC.9-12.G.MG.2** Apply concepts of density based on area and volume in modeling situations.

5. What is volume?

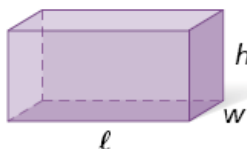
6. What is Cavalier's Principle?

### Volume of a Prism

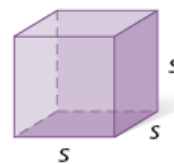
The volume of a prism with base area  $B$  and height  $h$  is  $V = Bh$ .



The volume of a right rectangular prism with length  $\ell$ , width  $w$ , and height  $h$  is  $V = \ell wh$ .



The volume of a cube with edge length  $s$  is  $V = s^3$ .



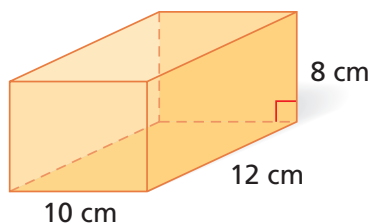
**Question:** What 3-D figure can cure step throat?

**Answer:** A penicillinder

## 1 Finding Volumes of Prisms

Find the volume of each prism. Round to the nearest tenth, if necessary.

**A**



$$V = \ell wh$$

$$= (10)(12)(8) = 960 \text{ cm}^3$$

*Volume of a right rectangular prism*

*Substitute 10 for  $\ell$ , 12 for  $w$ , and 8 for  $h$ .*

**B**

a cube with edge length 10 cm

$$V = s^3$$

$$= 10^3 = 1000 \text{ cm}^3$$

*Volume of a cube*

*Substitute 10 for  $s$ .*

Find the volume of each prism. Round to the nearest tenth, if necessary.

**C**

a right regular pentagonal prism with base edge length 5 m and height 7 m

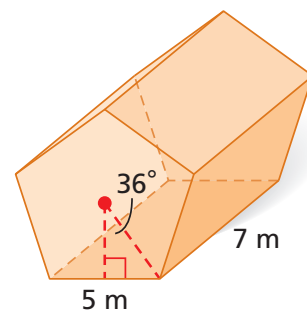
**Step 1** Find the apothem  $a$  of the base. First draw a right triangle on one base as shown. The measure of the angle with its vertex at the center is  $\frac{360^\circ}{10} = 36^\circ$ .

$$\tan 36^\circ = \frac{2.5}{a}$$

*The leg of the triangle is half the side length, or 2.5 m.*

$$a = \frac{2.5}{\tan 36^\circ}$$

*Solve for  $a$ .*



**Step 2** Use the value of  $a$  to find the base area.

$$B = \frac{1}{2}aP = \frac{1}{2}\left(\frac{2.5}{\tan 36^\circ}\right)(25) = \frac{31.25}{\tan 36^\circ} \quad P = 5(5) = 25 \text{ m}$$

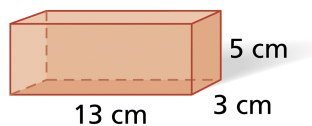
**Step 3** Use the base area to find the volume.

$$V = Bh = \frac{31.25}{\tan 36^\circ} \cdot 7 \approx 301.1 \text{ m}^3$$

"Envy is the art of counting the other fellow's blessings instead of your own."—  
*Humor Columnist, Harold Coffin*

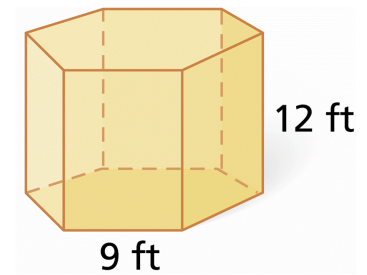
**Example 1. Find the volume of the prism. Round to the nearest tenth, if necessary.**

**A.**



**B.** Find the volume of a cube with edge length 15 in. Round to the nearest tenth, if necessary.

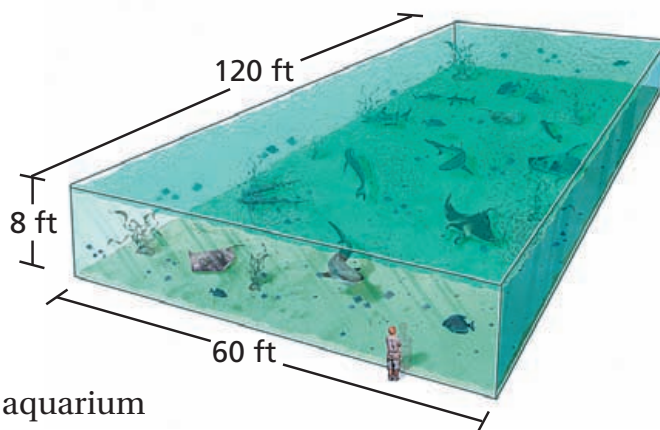
**C.** Find the volume of the right regular hexagonal prism.  
Round to the nearest tenth, if necessary.



**7. Guided Practice:** Find the volume of a triangular prism with a height of 9 yd whose base is a right triangle with legs 7 yd and 5 yd long.

## 2 Marine Biology Application

The aquarium at the right is a rectangular prism. Estimate the volume of the water in the aquarium in gallons. The density of water is about 8.33 pounds per gallon. Estimate the weight of the water in pounds.  
(Hint: 1 gallon  $\approx$  0.134  $\text{ft}^3$ )



**Step 1** Find the volume of the aquarium in cubic feet.

$$V = \ell wh = (120)(60)(8) = 57,600 \text{ ft}^3$$

**Step 2** Use the conversion factor  $\frac{1 \text{ gallon}}{0.134 \text{ ft}^3}$  to estimate the volume in gallons.

$$57,600 \text{ ft}^3 \cdot \frac{1 \text{ gallon}}{0.134 \text{ ft}^3} \approx 429,851 \text{ gallons} \quad \frac{1 \text{ gallon}}{0.134 \text{ ft}^3} = 1$$

**Step 3** Use the conversion factor  $\frac{8.33 \text{ pounds}}{1 \text{ gallon}}$  to estimate the weight of the water.

$$429,851 \text{ gallons} \cdot \frac{8.33 \text{ pounds}}{1 \text{ gallon}} \approx 3,580,659 \text{ pounds} \quad \frac{8.33 \text{ pounds}}{1 \text{ gallon}} = 1$$

The aquarium holds about 429,851 gallons. The water in the aquarium weighs about 3,580,659 pounds.



"I petitioned my owner for a dog house with greater volume."



"And this is what he built for me."

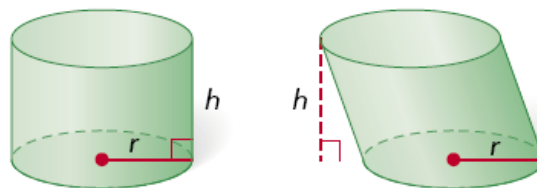
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**Example 2:** A swimming pool is a rectangular prism. Estimate the volume of water in the pool in gallons when it is completely full (Hint: 1 gallon  $\approx$  0.134 ft<sup>3</sup>). The density of water is about 8.33 pounds per gallon. Estimate the weight of the water in pounds.

**8. Guided Practice:** Estimate the volume in gallons and the weight of the water in the aquarium if the height were doubled.

### Volume of a Cylinder

The volume of a cylinder with base area  $B$ , radius  $r$ , and height  $h$  is  $V = Bh$ , or  $V = \pi r^2 h$ .



"Here's how I remember how to find the volume of any prism or cylinder."



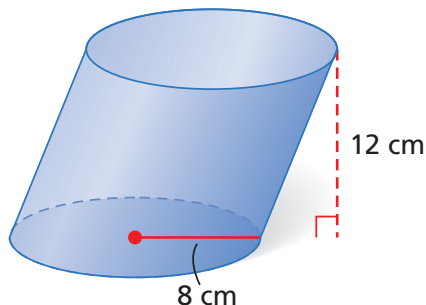
"Base times tall, will fill 'em all."

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### 3 Finding Volumes of Cylinders

Find the volume of each cylinder. Give your answers both in terms of  $\pi$  and rounded to the nearest tenth.

**A**



$$\begin{aligned}
 V &= \pi r^2 h && \text{Volume of a cylinder} \\
 &= \pi(8)^2(12) && \text{Substitute 8 for } r \text{ and 12 for } h. \\
 &= 768\pi \text{ cm}^3 \approx 2412.7 \text{ cm}^3
 \end{aligned}$$

**B** a cylinder with a base area of  $36\pi \text{ in}^2$  and a height equal to twice the radius

**Step 1** Use the base area to find the radius.

$$\begin{aligned}
 \pi r^2 &= 36\pi && \text{Substitute } 36\pi \text{ for the base area.} \\
 r &= 6 && \text{Solve for } r.
 \end{aligned}$$

**Step 2** Use the radius to find the height. The height is equal to twice the radius.

$$\begin{aligned}
 h &= 2r \\
 &= 2(6) = 12 \text{ cm}
 \end{aligned}$$

**Step 3** Use the radius and height to find the volume.

$$\begin{aligned}
 V &= \pi r^2 h && \text{Volume of a cylinder} \\
 &= \pi(6)^2(12) = 432\pi \text{ in}^3 && \text{Substitute 6 for } r \text{ and 12 for } h. \\
 &\approx 1357.2 \text{ in}^3
 \end{aligned}$$



"Dear Sir: Why do you sell dog food in tall cans and sell cat food in short cans?"

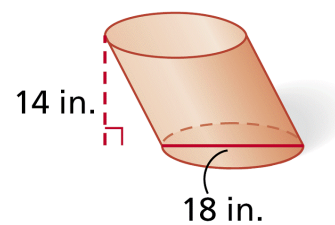


"Neither of these shapes is the optimal use of surface area when compared to volume."

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**Example 3.**

**A.** Find the volume of the cylinder. Give your answers in terms of  $\pi$  and rounded to the nearest tenth.



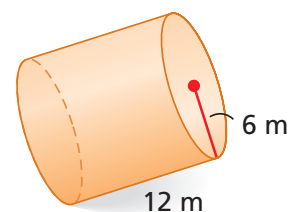
**B.** Find the volume of a cylinder with base area  $121\pi \text{ cm}^2$  and a height equal to twice the radius. Give your answer in terms of  $\pi$  and rounded to the nearest tenth.

**9. Guided Practice:** Find the volume of a cylinder with a diameter of 16 in. and a height of 17 in. Give your answer both in terms of  $\pi$  and rounded to the nearest tenth.

#### 4

### Exploring Effects of Changing Dimensions

The radius and height of the cylinder are multiplied by  $\frac{1}{2}$ . Describe the effect on the volume.



original dimensions:

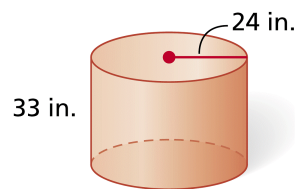
$$\begin{aligned} V &= \pi r^2 h \\ &= \pi(6)^2(12) \\ &= 432\pi \text{ m}^3 \end{aligned}$$

radius and height multiplied by  $\frac{1}{2}$ :

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi(3)^2(6) \\ &= 54\pi \text{ m}^3 \end{aligned}$$

Notice that  $54\pi = \frac{1}{8}(432\pi)$ . If the radius and height are multiplied by  $\frac{1}{2}$ , the volume is multiplied by  $\left(\frac{1}{2}\right)^3$ , or  $\frac{1}{8}$ .

**Example 4.** The radius and height of the cylinder are multiplied by  $\frac{2}{3}$ . Describe the effect on the volume.



**10. Guided Practice:** The length, width, and height of the prism are doubled. Describe the effect on the volume.

## 5

### Finding Volumes of Composite Three-Dimensional Figures

Find the volume of the composite figure.  
Round to the nearest tenth.

The base area of the prism is  $B = \frac{1}{2}(6)(8) = 24 \text{ m}^2$ .

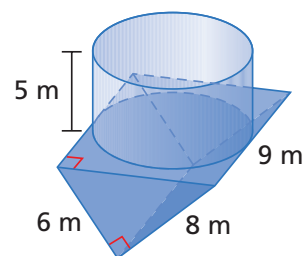
The volume of the prism is  $V = Bh = 24(9) = 216 \text{ m}^3$ .

The cylinder's diameter equals the hypotenuse of the prism's base, 10 m. So the radius is 5 m.

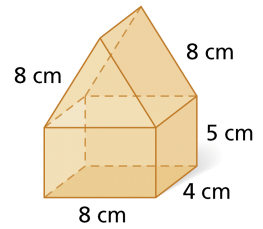
The volume of the cylinder is  $V = \pi r^2 h = \pi(5)^2(5) = 125\pi \text{ m}^3$ .

The total volume of the figure is the sum of the volumes.

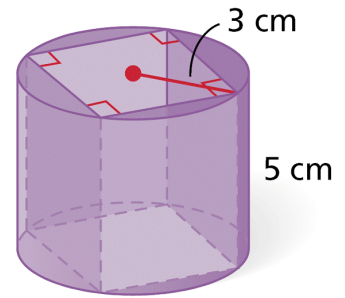
$$V = 216 + 125\pi \approx 608.7 \text{ m}^3$$



**Example 5:** Find the volume of the composite figure.



**11. Guided Practice:** Find the volume of the composite figure. Round the final answer to the nearest tenth.



**11-2 Volume of Prisms and Cylinders** (*p* 754) 13, 15, 16, 17, 19-23, 26, 34, 35, 36.

